IMITATION MODELLING AND COPULA METHOD USING FOR INVESTIGATION OF ECONOMIC PROBLEMS IN TRAINING PROCESS

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Introduction

The aim of investigation is presentation of using of multivariate copula method for modeling of economic systems in practice and in traning process. In real systems the parameters describing economic object not always have classical distribution. Between various random factors there is a nonlinear dependence which can't be modelled by means of linear correlation coefficient of Pearson. It causes of application of other methods of modeling. Recently the great popularity was received by modeling of joint distributions using copulas. Copula is a multidimensional function of distribution $C:[0,1]^n \rightarrow [0,1]$, which characterizes dependence between model factors. As a dependence measure between random factors can be used nonparametric correlation coefficient $\boldsymbol{\tau}$ by Kendall and $\boldsymbol{\rho}$ by Spearman.These functions have been introduced to model a joint distribution once the

Statistical modelling of logistics service costs by means of nonparametric methods

In the process of simulation modelling of logistics service costs the most frequently method used to model multivariate distribution incidental values is the parametric method of modelling. In this case it is necessary to establish parameters of common distribution of incidental values characterizing the factors under consideration. Usually this is done by means of evaluation of parameters of multivariate distribution, i.e. by establishing the most suitable distribution (copula), deriving from the available empirical data. When establishing the value of the price of delivery of cargo from the point of dispatch to the point of destination $\mathbf{P}_{delivery}$, such an approach will not be justified, since the amount of empirical information most frequently is insufficient for a credible assessment of parameters offered by the function of distribution. For resolving the particular problem it is necessary to use nonparametric modelling methods, given distribution of incidental values and then modelling parameters of distribution based on nonparametric values.

The value of $\mathbf{P}_{delivery}$ can be calculated as follows:

$$P_{deliv} = C_1 + C_2 + C_3 + C_4 + C_5, \qquad (1)$$

where $\mathbf{C_1}$ – cost of transportation of one loaded container from the port of loading to the port of discharge (unloading) (a port in the Republic of Latvia); $\mathbf{C_2}$ – the value of customs duties and charges and other compulsory payments necessary for moving the cargo on the territory of the Republic of Latvia; $\mathbf{C_3}$ – cost of handling cargo (loading-unloading operations) in the port; $\mathbf{C_4}$ – cost of services of the logistics firm ensuring delivery of cargo from the consignor to the consignee; $\mathbf{C_5}$ – cost of delivery of one loaded container from the port of discharge (unloading) to the port of destination of the cargo and transfer of this cargo to the consignee.

The copula for random values C1, C2,…, C5 can be described by equation (2):

$$C(u_1, u_2, ..., u_5) = \Phi(F_1^{-1}(x_1), F_2^{-1}(x_2), ..., F_5^{-1}(x_5)), \qquad (2)$$

where F_i – marginal distribution for random value C_i, i=1,2,…,5.

The algorithm of simulation of random n-dimensional vector $X=(X_1, X_2, \dots, X_n)$ is:

I. Simulate a variable X with distribution function G such that the Laplace transform of G is the inverse of the generator.

II. Simulate n independent variates V1,... , Vn.

III. Return $U = (\phi^{-1}(-\log(V1)/X), ..., (\phi^{-1}(-\log(Vn)/X))$.

Frank, Clayton and the Gumbel copula can be simulated using this procedure. For example for the Clayton copula simulation the algorithm is as follows:

I. Simulate a Gamma variate X~Gamma(1/θ,1)).

II. Simulate n independent standard uniform variates V1, ..., Vn.

III. Return U = $((1-\log(V1)/X)^{-1}/\theta,\ldots, (1-\log(Vn)/X)^{-1}/\theta))$.

The modelling of random vector $C=(C_1, C_2, C_3)$ has been realised by using of MatLab programm:

The algorithm of simulation of random vector $C=(C_1, C_2,\ldots, C_n)$ is:

MatLab kods:

```
n = 5000;
Rho = [1 -0.417 -0.522; -0.417 1 0.420; -0.522 0.42 1];
Z = mvnrnd([0 0 0], Rho, n);
U = normcdf(Z,0,1);
X = [logninv(U(:,1),4.75,1.32) logninv(U(:,2),2.53,0.55) wblinv(U(:,3),10.24,1.16)];
plot3(X(:,1),X(:,2),X(:,3),'.');
grid on; view([-50, 50]);
xlabel('Izm'); ylabel('NorIzm'); zlabel('Laiks');
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The illustration of the process of modelling of incidental value C=(C1, C2, C3) presented in Fig. 1.

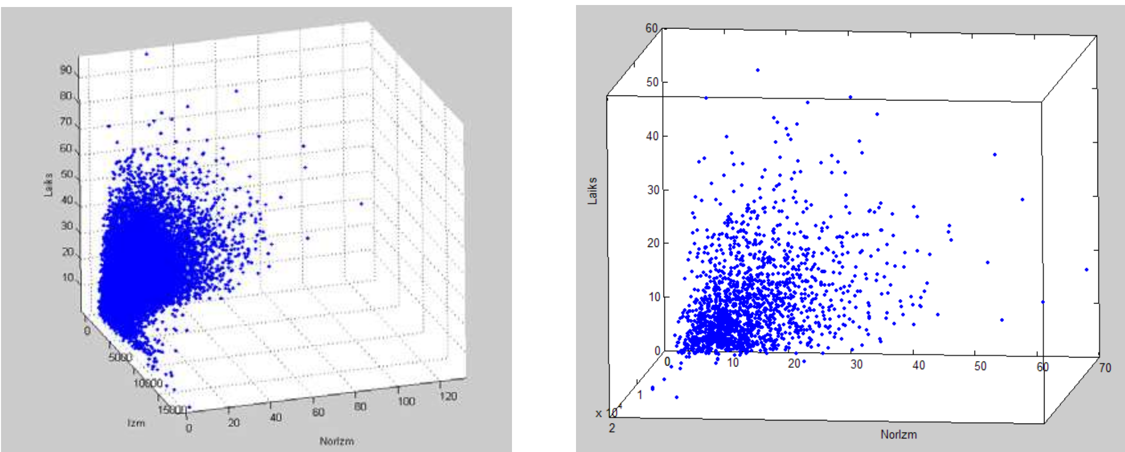

Fig. 1. Examples illustrating the process of modelling of incidental value C=(C1, C2, C3) with N=5000 and 50000 Monte-Carlo trials

By means of a histogram, marginal distributions are represented for constructing a copula, presenting a common distribution of factors. In the simplest case, distribution of each incidental value may be represented by means of a nonparametric method – a block chart. For example, Fig. 2 shows how, derived from the block charts of distribution of two incidental values – factors $\mathbf{X_1}$ and $\mathbf{X_2}$, it is possible to model a bivariate common distribution considering the dependence between the factors.

The experience of modelling by using nonparametric methods demonstrates sufficiently good approximation possibilities and comparative simplicity of implementation of this method for modelling $P_{delivery}$. The quality of approximation has been evaluated by using of χ^2 method.

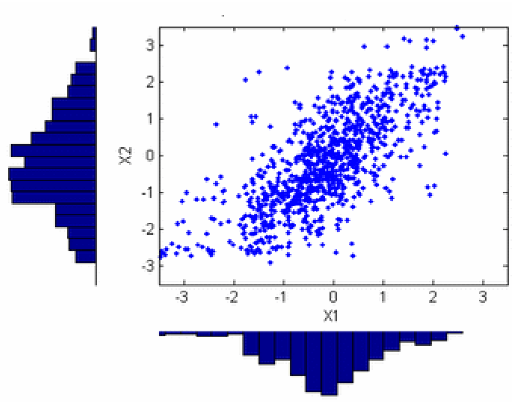

Fig. 2. Example illustrating the process of modelling of bivariate incidental value, based on nonparametric evaluation of their distribution – the histograms

Stochastic modelling of the stability of economic system

The primary goal of management of economic system (ECSYS) stability is maintaining the criterion of stability with regard to the given interval of allowable values of the chosen criterion of stability. For the evaluation of ECSYS stability it is offered to use:

– complex variable C^F of the financial stability of ECSYS;

– complex indicator C^Q of the production stability of ECSYS;

– integral indicator of the stability of the system J, which is the function of several variables and is aggregating the complex evaluation indicators C^F, C^Q of the ECSYS stability.

Criteria for estimation of ECSYS stability. The paper supposes that ECSYS is a logistics system. Modelling of the stability of functioning of a real logistics system necessitates the transfer from a continuous ECSYS functioning model to a discrete model. In a discrete model time moments are determined when the information of the condition of ECSYS stability is received. With the improvement of information supply of ECSYS performance, a real possibility arises for choosing the optimal order of control moments of time t_0, t_1, t_2, … , t_n of the information received about the condition of ECSYS stability. The process of modelling of ECSYS stability is presented (see Fig. 3).

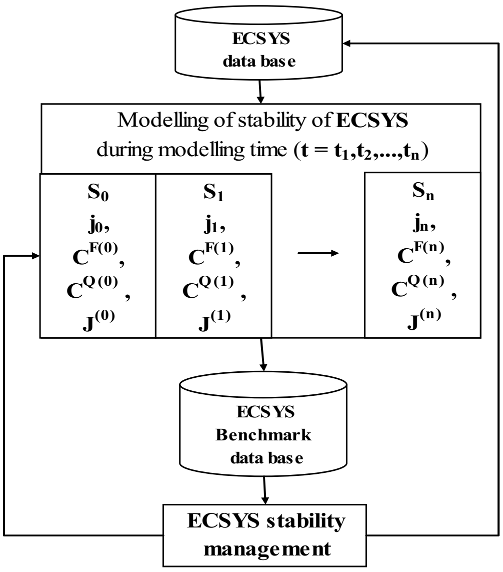

Fig. 3. Process of modelling of ECSYS stability

In Fig. 3 S_i – state of ECSYS in moments of time t_i; i = 0, 1,…, n); n – number of states of ECSYS; j – index of the participant of the logistics process; j_i – number of participants of the logistics process at the moment of time t_i, (j = 0, 1, … , m); $J^{(i)}$ – integrated criterion of the system stability at the moments of time t_i, (i = 0, 1, …, n).

The complex criterion of financial stability ECSYS $C^{F(j)}$ is the financial stability of j-th participant of ECSYS taking part in the logistics process at the moment of time t_i. Under favourable conditions of ECSYS performance at the moment of time t_i, in the system the issue of insolvency of ECSYS participants should not arise. That is the balance (B_{ij}) of any ECSYS participant at the moment of time t_i is a positive value:

$$B_{i,j} > 0\,. \tag{3}$$

The complex criterion of the financial stability of ECSYS may be presented as a function:

$$C^{F(i)} = f_1(V_{(-)}^{i,j}, n_{(-)}^{i,j}, \omega) \;, \tag{4}$$

where $C^{F(i)}$ – complex criterion of financial stability of ECSYS at the time moment t_i; – $V^{ij}_{(-)}$ – normalized criterion of the insolvency of j-th ECSYS participant at the time moment t_i, should satisfy the inequalities:

$$-1 \le V_{(-)}^{i,j} \le 0\,; \tag{5}$$

$n^{ij}_{(-)}$- number of insolvent ECSYS participants at the time moment t_i; w – random factor.

Modelling of financial stability is directed towards defining the value of the "zones of risk" when the logistics system for meeting financial liabilities should use financial reserves, as well as for finding the point of first time of no negativity of the financial flow. The process of modelling of the financial flow during the modelling time is described in detail in papers. In real logistics systems there are problems with holding payments in the case of transactions between ECSYS participants. The breach of the contract and/or partial fulfilment of financial contracts by one of the participants of ECSYS lead to breaking of the financial stability of the whole system. Thus, it is offered to create and use the financial reserves of the system (Res^{ij}**)** for ensuring the financial stability of ECSYS. The complex criterion of financial stability of ECSYS, taking into account the financial reserves $C_{Res}{}^{F(i)}$, may be presented as a function:

$$C_{\mathrm{Re}s}{}^{F(i)} = f_1(V_{(-)}^{i,j}, n_{(-)}^{i,j}, \omega, \mathrm{Re}\,s^{i,j}) \;, \tag{6}$$

where R_i – the total amount of resources in ECSYS at the moment of time t_i.

The modelling process of ECSYS stability is implemented applying a set of alternative strategies of ECSYS performance by using the dynamic programming and benchmarking method. The introduction of a set of alternative strategies supports stable functioning of ECSYS in the conditions of uncertainty. Thus, the integrated criterion $J^{(i)}$ changes its values in the feasible region:

$$\min J_j{}^i(t) \le \sum J_j{}^i(t) \le \max J_j{}^i(t)\,. \tag{7}$$

The change of criteria of ECSYS stability is a signal for adjusting the functioning strategy of ECSYS. It means the necessity arises to implement another strategy which may be selected from the set of alternative strategies.

Practical use of statistical modelling

The graphic illustration of the first time no negativity is presented in Fig. 4.

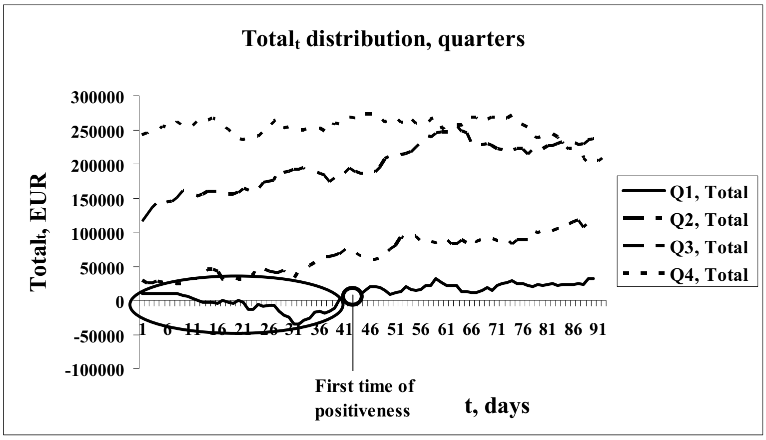

Fig. 4. Scheme of first time of no negativity

The following parameters are shown in Fig. 4:

– first time of no negativity;

– ”zone of risk” of the financial stability of economic system.

The results of modelling allow to identify ”zones of risk” of the financial stability of any ECSYS participant.

On the basis of the information available it is possible to consider:

a) modelling of financial stability of ECSYS in conditions of risk or uncertainty;

b) identification of ”zones of risk ” in management of financial stability of ECSYS in conditions of risk or uncertainty by Monte-Carlo method. For modelling the financial stability of ECSYS the following variables are used:

ω_{1i} – the period of time in days between actual (modelled) term of receipt of payment on the account of firm (**T₂**) from the **i-**th consignor (consignee) of the cargo and planned (in accordance with the contract terms signed) term of receipt of payment on the account of EC-SYS for the cargo transportation and registration services from the point of dispatch to the point of destination (**T₁**);

ω_{2j} – the period of time in days between the modelled term of payment by the ECSYS the account to the **j-**th participant of ECSYS and planned (in accordance with the contract terms signed) term of payment to the **j-**th participant of ECSYS for the services rendered.

Conclusion

The modern economic analysis basing on the using of information technologies shows that in the real systems the parameters describing the economic objects, not always have the Gauss distribution. The nonlinear dependence exists between various factors. In these cases it is impossible to use the linear correlation coefficient for evaluation of measure of dependences between factors. It requires using another method for evaluation the measure of dependences between factors.

In our days, designing real economic systems very much popular is becoming the use of copulas, which fully characterizes the nonlinear connection between main factors of the model and allows uniting margin functions into multivariate distribution function. The application of modelling is connected with the fact that frequently it is not possible to provide a definite description of the behaviour of the economic system being investigated.

The introduction of a set of alternative strategies supports stable functioning of economic system in the conditions of risk or uncertainty. By using Monte Carlo method and multivariate copula method for modelling it has become possible:

• to set alternative strategies of economic system performance;

• to combine different marginal distributions using copula method;

• to model economic system stability using multimodal and nonparametric technique;

• to model the "risk zones" in which the stability of economic system has been distorted;

• to manage functioning of economic system.

The theoretical and practical results obtained as a result of this research can be applied in practical activities of companies and in training process.

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